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QUARK FAMILY DISCRIMINATION AND FLAVOUR-CHANGING NEUTRAL CURRENTS IN THE $\mathbf{SU(3)_C \otimes SU(3)_L \otimes U(1)}$ MODEL WITH RIGHT-HANDED NEUTRINOS

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Abstract

Contributions of flavour-changing neutral currents in the 3 3 1 model with right-handed neutrinos to mass difference of the neutral meson system $\Delta m_P(P = K, D, B)$ are calculated. Using the Fritzsch anzats on quark mixing, we show that the third family should be different from the first two. We obtain a lower bound on mass of the new heavy neutral gauge boson as 1.02 TeV.

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I. Introduction

The standard model (SM) has been very successful in explaing high energy phenomena. However there still remain some important questions we should understand. It is especially necessary to answer the quark family problem and hierarchy puzzle. In addition, the SuperKamiokande atmospheric neutrino data [1] provided an evidence for neutrino oscillation and consequently non-zero neutrino mass. It is known that, neutrinos are massless in the SM, therefore the SuperKamiokande result calls firstly for the SM extension.

Among the possible extensions, the models based on the $\mathrm{SU(3)}_C \otimes \mathrm{SU(3)}_L \otimes \mathrm{U(1)}_N$ (3 3 1) gauge group [2, 3] have some interesting properties such as: first, it can explain why family number N is equal to three. Second, one quark family is treated differently from the other two, and this gives some indication as to why the top quark is unbalancing

heavy. Third, the Peccei-Quinn symmetry, necessary to solve the strong-CP problem, follows naturally from particle content in these models [4].

There are two main versions of 3 3 1 models: the minimal [2, 5] in which all lepton components $(\nu, l, (l_L)^c)$ belong to the lepton triplet and a variant, in which right-handed neutrinos (r.h.neutrinos) are included i.e. $(\nu, l, (\nu)^c_L)$ (hereafter we call it the model with r.h. neutrinos).

The fact that one quark family is treated differently from the other two, leads to the flavour-changing neutral currents (FCNC's), which give a contribution to the mass difference of the neural meson systems at the tree level. The effect in the minimal model was considered in [6]. In this paper we shall consider the FCNC's effect in the model with r.h. neutrinos.

II. The model

Let us briefly recapitulate the basic elements of the model. Fermions are in triplet

$$f^a_L = \begin{pmatrix} \nu^a_L \\ e^a_L \\ (\nu^c_L)^a \end{pmatrix} \sim (1, 3, -1/3), e^a_R \sim (1, 1, -1), \tag{1}$$

where a = 1, 2, 3 is the family index.

Two of the three quark families transform identically and one family (it does not matter which one) transforms in a different representation of the gauge group $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$:

$$Q_{iL} = \begin{pmatrix} d_{iL} \\ -u_{iL} \\ D_{iL} \end{pmatrix} \sim (3, \bar{3}, 0), \tag{2}$$

$$u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), D_{iR} \sim (3, 1, -1/3),\ i = 1, 2,$$

$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix} \sim (3, 3, 1/3),$$

$$u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_R \sim (3, 1, 2/3),$$

where D and T are exotic quarks with electric charges $-\frac{1}{3}$ and $\frac{2}{3}$, respectively.

Fermion mass generation and symmetry breaking can be achieved with just three $SU(3)_L$ triplets

$$\chi = \begin{pmatrix} \chi^o \\ \chi^- \\ \chi^{,o} \end{pmatrix} \sim (1, 3, -1/3), \rho = \begin{pmatrix} \rho^+ \\ \rho^o \\ \rho^{,+} \end{pmatrix} \sim (1, 3, 2/3),\ \eta = \begin{pmatrix} \eta^o \\ \eta^- \\ \eta^{,o} \end{pmatrix} \sim (1, 3, -1/3). \tag{3}$$

All the Yukawa terms of quarks are given

$$
\begin{aligned}
\mathcal{L}^{\chi}_{Yuk} &= \lambda_1 \bar{Q}_{3L} T_R \chi + \lambda_{2ij} \bar{Q}_{iL} d^{'}_{jR} \chi^* + \text{h.c.} \\
&= \lambda_1 (\bar{u}_{3L} \chi^o + \bar{d}_{3L} \chi^- + \bar{T}_L \chi^{,o}) T_R + \lambda_{2ij} (\bar{d}_{iL} \chi^{o*} - \bar{u}_{iL} \chi^+ + \bar{D}_{iL} \chi^{,o*}) D_{jR} + \text{h.c.} \\
\mathcal{L}^{\eta}_{Yuk} &= \lambda_{3a} \bar{Q}_{3L} u_{aR} \eta + \lambda_{4ia} \bar{Q}_{iL} d_{aR} \eta^* + \text{h.c.} \\
&= \lambda_{3a} (\bar{u}_{3L} \eta^o + \bar{d}_{3L} \eta^- + \bar{T}_L \eta^{,o}) u_{aR} + \lambda_{4ia} (\bar{d}_{iL} \eta^{o*} - \bar{u}_{iL} \eta^+ + \bar{D}_{iL} \eta^{,o*}) d_{aR} + \text{h.c.} \\
\mathcal{L}^{\rho}_{Yuk} &= \lambda_{1a} \bar{Q}_{3L} d_{aR} \rho + \lambda_{2ia} \bar{Q}_{iL} u_{aR} \rho^* \\
&= \lambda_{1a} (\bar{u}_{3L} \rho^+ + \bar{d}_{3L} \rho^o + \bar{T}_L \rho^{,+}) d_{aR} + \lambda_{2ia} (\bar{D}_{iL} \rho^- - \bar{u}_{iL} \rho^{o*} \\
&\quad + \bar{D}_{iL} \rho^{,-}) u_{aR} + \text{h.c.}. \qquad (4)
\end{aligned}
$$

In this model the Higgs triplets in Eq. (3) should develop VEVs as follow:

$$\langle \chi \rangle^T = (0, 0, \omega/\sqrt{2}),\ \langle \rho \rangle^T = (0, u/\sqrt{2}, 0),\ \langle \eta \rangle^T = (v/\sqrt{2}, 0, 0).$$

The new complex gauge bosons in this model are $\sqrt{2} X^0_\mu = W^4_\mu - i W^5_\mu, \sqrt{2} Y^+_\mu = W^6_\mu - i W^7_\mu$. Both these bosons carry lepton number two, hence they are called bileptons. In [7] the first constraints on masses of the bileptons in this model are derived by considering S, T parameters: 213 GeV $\leq M_{Y^+} \leq$ 234 GeV, 230 GeV $\leq M_{X^0} \leq$ 251 GeV.

The physical neutral gauge bosons are mixtures of Z, Z':

$$
\begin{aligned}
Z^1 &= Z \cos\phi - Z' \sin\phi, \\
Z^2 &= Z \sin\phi + Z' \cos\phi, \qquad (5)
\end{aligned}
$$

where the photon field A_μ and Z, Z' are given by:

$$
\begin{aligned}
A_\mu &= s_W W^3_\mu + c_W \left(-\frac{t_W}{\sqrt{3}}\, W^8_\mu + \sqrt{1 - \frac{t_W^2}{3}}\, B_\mu \right), \\
Z_\mu &= c_W W^3_\mu - s_W \left(-\frac{t_W}{\sqrt{3}}\, W^8_\mu + \sqrt{1 - \frac{t_W^2}{3}}\, B_\mu \right), \qquad (6) \\
Z'_\mu &= \sqrt{1 - \frac{t_W^2}{3}}\, W^8_\mu + \frac{t_W}{\sqrt{3}}\, B_\mu.
\end{aligned}
$$

Here s_W stands for $\sin\theta_W$. The mixing angle ϕ is defined by

$$\tan^2\phi = \frac{m^2_Z - m^2_{Z^1}}{M^2_{Z^2} - m^2_Z}, \qquad (7)$$

where m_{Z^1} and M_{Z^2} are the *physical* mass eigenvalues.

The interactions among fermions and Z_1, Z_2 are given as follows:

$$
\begin{aligned}
\mathcal{L}^{NC} &= \frac{g}{2c_W} \left\{ \bar{f} \gamma^\mu [a_{1L}(f)(1-\gamma_5) + a_{1R}(f)(1+\gamma_5)] f Z^1_\mu \right. \\
&\quad \left. + \bar{f} \gamma^\mu [a_{2L}(f)(1-\gamma_5) + a_{2R}(f)(1+\gamma_5)] f Z^2_\mu \right\}. \qquad (8)
\end{aligned}
$$

where

$$
\begin{aligned}
a_{1L,R}(f) &= \cos\phi\ [T^3(f_{L,R}) - s_W^2 Q(f)] \\
&\quad -c_W^2 \left[\frac{3N(f_{L,R})}{(3-4s_W^2)^{1/2}} - \frac{(3-4s_W^2)^{1/2}}{2c_W^2} Y(f_{L,R})\right] \sin\phi, \\
a_{2L,R}(f) &= c_W^2 \left[\frac{3N(f_{L,R})}{(3-4s_W^2)^{1/2}} - \frac{(3-4s_W^2)^{1/2}}{2c_W^2} Y(f_{L,R})\right] \cos\phi \\
&\quad + \sin\phi\ [T^3(f_{L,R}) - s_W^2 Q(f)].
\end{aligned} \tag{9}
$$

Here $T^3(f)$ and $Q(f)$ are, respectively, the third component of the weak isospin and the charge of the fermion f.

III. Flavour-changing neutral currents and mass difference of the neutral meson systems

Due to the fact that one family of left-handed quarks is treated differently from the other two, the N charges for left-handed quarks are different too (see Eq. (2)). Therefore flavour-changing neutral currents Z_1, Z_2 occur through a mismatch between weak and mass eigenstates.

Let us diagolize mass matrices by three biunitary transformations

$$
\begin{aligned}
U'_L &= V_L^U U_L,\ U'_R = V_R^U U_R, \\
D'_L &= V_L^D D_L,\ D'_R = V_R^D D_R,
\end{aligned} \tag{10}
$$

where $U \equiv (u,c,t)^T,\ D \equiv (d,s,b)^T$.
The usual Cabibbo-Kobayashi-Maskawa matrix is given by

$$
V_{CKM} = V_L^{U+} V_L^D. \tag{11}
$$

Using unitarity of the V^D and V^U matrices, we get flavour-changing neutral interactions

$$
\begin{aligned}
\mathcal{L}_{ds}^{NC} &= \frac{g c_W}{2\sqrt{3-4s_W^2}} \left[V_{Lid}^{D*} V_{Lis}^D\right] \bar{d}_L \gamma_\mu s_L \left(\cos\phi Z_2^\mu - \sin\phi Z_1^\mu\right), \\
\mathcal{L}_{uc}^{NC} &= \frac{g c_W}{2\sqrt{3-4s_W^2}} \left[V_{Liu}^{U*} V_{Lic}^U\right] \bar{u}_L \gamma_\mu c_L \left(\cos\phi Z_2^\mu - \sin\phi Z_1^\mu\right), \\
\mathcal{L}_{db}^{NC} &= \frac{g c_W}{2\sqrt{3-4s_W^2}} \left[V_{Lid}^{D*} V_{Lib}^D\right] \bar{d}_L \gamma_\mu b_L \left(\cos\phi Z_2^\mu - \sin\phi Z_1^\mu\right),
\end{aligned} \tag{12}
$$

where i denotes the number of "different" quark family i.e. the $\mathrm{SU}(3)_L$ quark triplet.

For the neutral kaon system, we get then effective Lagrangian

$$
\mathcal{L}_{eff}^{\Delta S=2} = \frac{\sqrt{2} G_F\ c_W^4 \cos^2\phi}{(3-4s_W^2)} \left[V_{Lid}^{D*} V_{Lis}^D\right]^2 |\bar{d}_L \gamma^\mu s_L|^2 \left(\frac{m_{Z_1}^2}{M_{Z_2}^2} + \tan^2\phi\right). \tag{13}
$$

Similar expressions can be easily written out for $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$ systems. From (13) it is straightforward to get the mass difference

$$\Delta m_P = \frac{4G_F\ c_W^4\ \cos^2\phi}{3\sqrt{2}(3-4s_W^2)} \left[V_{Lid}^{D*} V_{Li\alpha}^{D}\right]^2 \left(\frac{m_{Z_1}^2}{M_{Z_2}^2} + \tan^2\phi\right) f_P^2 B_P m_P, \tag{14}$$

where $\alpha = s$ for the $K_L - K_S$ and $\alpha = b$ for the $B^0 - \bar{B}^0$ mixing systems. The $D^0 - \bar{D}^0$ mass difference is given by the expression for the K^0 system with replace of V^D by V^U. The $Z - Z'$ mixing angle ϕ was bounded and to be [3, 5]: $|\phi| \leq 10^{-3}$, hence if M_{Z_2} is in order of one hundred TeV, the $Z - Z'$ mixing has to be taken into account.

In the usual case, the $Z - Z'$ mixing is constrained to be very small, it can be safely neglected. Therefore FCNC's occur only via Z_2 couplings. For the shothand hereafter we rename Z_1 to be Z and Z_2 to be Z'.

Since it is generally recognized that the most stringent limit from Δm_K, we shall mainly discuss this quantity. We use the experimental values [8]

$$\Delta m_K \ = \ (3.489 \pm 0.009) \times 10^{-12}\ \text{MeV}, \qquad m_K \simeq 498\ \text{MeV} \tag{15}$$

and

$$\sqrt{B_K} f_K \ = \ 135 \pm 19\ \text{MeV}. \tag{16}$$

Following the idea of Gaillard and Lee [9], it is reasonable to expect that Z' exchange contributes a Δm no larger than observed values. Substituting (15) and (16) into (14) we get

$$M_{Z'} \ > \ 2.63 \times 10^5\ \eta_{Z'} \left[Re(V_{Lid}^{D*} V_{Lis}^{D})^2\right]^{1/2}\ \text{GeV}. \tag{17}$$

where $\eta_{Z'} \approx 0.55$ is the leading order QCD corrections [10].

Let us call $\Delta m_K^{min}, \Delta m_K^{rhn}$ contributions to Δm from the Z' in the minimal 3 3 1 model and in the model with r.h. neutrinos, respectively. We have then

$$R \equiv \frac{\Delta m_K^{min}}{\Delta m_K^{rhn}} = \frac{2(3-4s_W^2)}{3(1-4s_W^2)} = 19.7, \tag{18}$$

for [8] $s_W^2 = 0.2312$. Because of the denominator, the relation is highly sensitive to the value of the Weinberg angle. It is easy to see that a limit for the Z' following from Eq (17) in the model with r.h.neutrinos is approximately 4.4 times smaller than that in the minimal version.

From the present experimental data we cannot get the constraint on $V_{Lij}^{U,D}$. These matrix elemetns are only constrainted by (11). However, it would seem more natural, if Higgs scalars are associated with fermion generations, to have the choice of nondiagonal

elements depends on the fields to which the Higgs scalars couple. By this way, the simple Fritzsch [11] scheme gives us

$$V_{ij}^D \approx \left(\frac{m_i}{m_j}\right)^{1/2}, \qquad i < j. \tag{19}$$

Combining (17) and (19) we get the following bounds on $M_{Z'}$:

$$\begin{aligned} M_{Z'} &\geq 38 \text{ TeV}, \quad \text{if the first or the second quark family is different (in triplet)} \\ M_{Z'} &\geq 1.02 \text{ TeV}, \quad \text{if the third quark family is different} \end{aligned} \tag{20}$$

From (20) we see that to keep relatively low bounds on $M_{Z'}$ the third family should be the one that is different from the other two i.e. is in triplet.

Our numerical estimation is based on the fact that all the phases of the matrix elements equal to zero. The inclusion of complex phases would induce to a reduction in the mass limit. However the hierarchical picture should not be modified.

IV. Summary

We have studied the FCNC's in the 3 3 1 model with r.h. neutrinos arisen from the family discrimination in this model. This gives a reason to conclude that the third family should be treated differently from the first two. In this sense, the Δm_K gives us the lower bound on $M_{Z'}$ as 1.02 TeV. It is to be mentioned that our conclusion is similar with that in the minimal version [12]:

$$\begin{aligned} M_{Z'} &\geq 315 \text{ TeV}, \quad \text{if the first or the second quark family is different (in triplet)} \\ M_{Z'} &\geq 10 \text{ TeV}, \quad \text{if the third quark family is different} \end{aligned}$$

It is interesting to note that in the Fritzsch anzats, the limits for $M_{Z'}$ following from Δm_B are independent of the family choice.

We emphasize that the FCNC's in the minimal model are larger than those in the considered version due to the factor $\frac{1}{(1-4s_W^2)}$. Therefore the lower bounds on $M_{Z'}$ are smaller accordingly.

In both versions of the 3 3 1 models, the third quark family should be different from the first two, and this gives us some indication of why the top quark is so heavy.

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